Large time behavior of the coupled systems of two Korteweg-de Vries equations

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Abstract:

In this talk, we present some recent results related to Large time behaviour of the lower and higher order coupled systems of two Korteweg-de Vries equations under the effect of some boundary dissipation posed on a interval with some boundary conditions, [4, 5].

This kind of systems are know as a family of Boussinesq systems of KdV-Type and theis has been proposed in [2, 3, 6] to describe the bi-directional propagation of small amplitude long waves on the surface of shallow water.

We consider the following systems introduced in [2, 3] and [6], respectively,

$$\begin{cases} \eta_t + w_x + w_{xxx} + (\eta w)_x = 0, & \text{in } (0,L) \times (0,+\infty), \\ w_t + \eta_x + \eta_{xxx} + w w_x = 0, & \text{in } (0,L) \times (0,+\infty), \\ \eta(x,0) = \eta_0(x), \quad w(x,0) = w_0(x), & \text{in } (0,L), \end{cases} \tag{1}$$

and

$$\begin{cases} \eta_t + u_x - a u_{xxx} + a_1(\eta u)_x + a_2(\eta u_{xx})_x + b u_{xxxxx} = 0, & \text{in } (0,L) \times (0,+\infty), \\ u_t + \eta_x - a \eta_{xxx} + a_1 u u_x + a_3(\eta \eta_{xx})_x + a_4 u_x u_{xx} + b \eta_{xxxxx} = 0, & \text{in } (0,L) \times (0,+\infty), \end{cases} \tag{2}$$

with some relations between the coefficients and some boundary conditions.

Firstly, we deal with the local rapid exponential stabilization for a Boussinesq system of KdV-KdV type (1) with Diriclhet-Neumann boundary condition. Our main result for the system (2) is to design a parameter family of feedback laws for which the solution of the associated linearized system are exponentially decreasing in the energy space.

Work in collaboration with A.F. Pazoto (UFRJ) and R.A. Capistrano-Filho (UFPE)

References

[1] J. V. Boussinesq, *Théorie générale des mouvements qui sont propagés dans un canal rectangulaire horizontal.* C. R. Acad. Sci. Paris **72** (1871), 755–759.

[2] J. L. Bona, M. Chen and J. C. Saut, *Boussinesq equations and other systems for small-amplitude long waves in nonlinear dispersive media I: Derivation and linear theory.* Journal of Nonlinear Science **12** (2002), 283–318.

[3] J. L. Bona, M. Chen and J. C. Saut, *Boussinesq equations and other systems for small-amplitude long waves in nonlinear dispersive media: II. The nonlinear theory.* Nonlinearity **17** (2004), 925–952.

[4] R.A. Capistrano-Filho and F.A. Gallego, Asymptotic behavior of Boussinesq system of KdV-KdV type. *Journal of Differential Equation (2018). Vol 265, Issue 6, Pages 2341-2374.*

[5] R.A. Capistrano-Filho, F.A. Gallego and A.F. Pazoto, On the well posedness and large-time behavior of higher order Boussinesq system. *arXiv preprint ArXiv:1712.08452, (2018)*, Submitted.

[6] P. J. Olver, *Hamiltonian and non-Hamiltonian models for water waves.* Lecture Notes in Physics, vol.195, Springer Verlag, (1984), 273—296.